

## Official Lesson Plan Form

Lesson Title: Intro to Complex Variables Date: \_\_\_\_\_Subject/Class: Mathematics: Pre-Calculus Grade Level: 11 / 12

MI Framework Standard(s) [(+) indicates an AP standard]:

N-CN:

Extending 1, 2, 3. (+)

F-IF:

1. Analyze functions using different representations
  - a. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
2. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Objectives:

Upon completion of the lesson, students will be able to...

1. Use Euler's identities to convert expressions with complex variables between trigonometric, exponential, and rectangular forms.
2. Derive and prove various useful identities from real-number mathematics using complex numbers.
3. Graph the real part, imaginary part, and phase portrait of complex variable functions and explain what each graph represents as well as its limitations.

Rationale for the Lesson:

In this lesson, students will be introduced to the basics of complex variables and complex analysis. This goes somewhat beyond the standards but will be useful in expanding the way students look at complex numbers. Additionally, students going into higher mathematics in college will benefit significantly from this introduction as analysis is one of the main branches of mathematics.

Student Prior Knowledge/Common Misconceptions:

Prior experience with complex numbers from the previous lessons is required, as is generally good mathematical knowledge (trig, algebra, exponents, and intuition). Good proof skills are helpful though not required. Working with complex variables and analysis is not easy, and there are many ways for students to go wrong. In particular, it's just difficult to visualize these incredibly abstract concepts.

Materials and resources needed for lesson:

- Whiteboard, markers
- Worksheets / example problems (self-made if time allows)
- Paper/pencil
- Computer with graphing program installed such as MATLAB, Mathematica

References, when appropriate:

## Opening

Time	Procedures/Details
5m	Warm up discussion: Taylor series absolute basics – how they work and that there are known series for standard functions. Don't go into the math behind why they work; that's for later after derivatives. Alternatively, if this is in a Calc class, just a warm up/reminder on what Taylor series are.
5m [10m total]	
	Any questions from the homework/project?

## Instructional Activities

Time	Procedures/Details
20m	Derive Euler's identity $e^{i\pi} + 1 = 0$ . Run through from extending the Taylor series of $e^x$ to $e^z$ , then breaking that up into $e^x e^{iy}$ , then to the Taylor series of sin and cos, to a graphical connection with the polar form we already know, finally substituting in $\pi$ for $\theta$ and you get the identity.
10m	
[30m total]	
	Use this to manipulate weird things like $i^i$ and show how this identity of $e^{i\theta}$ is an incredibly useful tool for manipulating complex numbers. Provide additional video resources involving a proof of Euler's identity and basic group theory (see teacher notes).

## Closing

Time	Procedures
10m	Show graphs of functions on the complex plane as a taste of just how crazy these things can get. Use Matlab to demonstrate Re, Im, and phase portraits of functions from the audience; discuss singularities and other neat features. Either prepare for the test and presentations if done beforehand or the next unit if done afterwards. Collect any outstanding homework.
[10m total]	

Assessment (including diagnostic, formative and/or summative):

(Diagnostic/formative)

Questions / observations during class:

- This is going to be a tough one to understand and visualize in class – ideally stop every so often and see if students understand what's going on or if they're totally lost. Nothing really specific beyond that.

(Summative)

Proof Project:

See lesson plan 4 for the proof project and rubric (attached there).

(Summative)

Unit test, attached.

Additional Teacher Notes:

Additional video resources regarding Euler's identity:

[https://www.youtube.com/watch?v=F\\_0yfvM0UoU](https://www.youtube.com/watch?v=F_0yfvM0UoU) (~6 minutes but leaves some things out)

<https://www.youtube.com/watch?v=mvmuCPvRoWQ> (~25 minutes, goes into group theory, better presentation)