Michigan Technological University **Division of Teacher Education**

Official Lesson Plan Form

Lesson Title: Polynomial Equations and Complex Roots Date: _____

Subject/Class: Mathematics: Pre-Calculus Grade Level: 11 / 12

MI Framework Standard(s) [(+) indicates an AP standard]:
N-CN:

3. Use complex numbers in polynomial identities and equations.
   a. Solve quadratic equations with real coefficients that have complex solutions.
   b. (+) Extend polynomial identities to the complex numbers. For example, rewrite x^2+4 as (x +2i)(x –2i).
   c. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Objectives:
Upon completion of the lesson, students will be able to...

1. Solve polynomial equations with real coefficients that have complex solutions.
2. Describe the Fundamental Theorem of Algebra and apply it to polynomial equations to find complex solutions.
3. Explain which algebraic identities apply to complex numbers and which require tweaking.
4. Apply complex numbers to real world problems.

Rationale for the Lesson:
In this lesson, students will apply the understanding of complex numbers they have built over the previous lessons to polynomial equations. The Fundamental Theorem of Algebra will be discussed and used to find polynomial roots. Through this lesson, students will gain a deeper understanding of the previously fully-real world of polynomial equations and will grasp a more complete set of rules.

Student Prior Knowledge/Common Misconceptions:
Students should have a fairly solid knowledge of working problems with complex numbers involved and converting between various forms (Cartesian and Polar at this point, from the previous two lessons). Somewhat advanced manipulations of polynomials and related identities will be helpful as well, though as long as the basic understanding of polynomial equations is present, it'll be fine. Misconceptions may include dismissing imaginary solutions inadvertently through things such as z^7=128 leading to just z=2 rather than recognizing there must be seven solutions.

Materials and resources needed for lesson:

- Whiteboard, markers
- Worksheets / example problems (self-made if time allows)
- Paper/pencil
- Helpful: computer with graphing program installed such as MATLAB, Mathematica

References, when appropriate:

Opening

Time	Procedures/Details
5m 5m [10m total]	Warm up problem: Find the roots of a quadratic equation (in this case will both be complex roots). Any questions from the homework?

Instructional Activities

Time	Procedures/Details
10m 10m 20m [40m total]	Introduce another quadratic equation and ask students to predict how many roots and of what type there will be. Go over why they're making whatever prediction, solve and see if they're correct. Introduce a cubic equation and ask again. Solve and reflect. Ask students if they can predict how many roots and what type will be the result of any polynomial with real coefficients (and why). Go over the Fundamental Theorem of Algebra and why it works. Similarly, go over conjugate pairs and determine why there can only be conjugate pairs of complex solutions if the coefficients are real. (hence a quadratic, for example, will always have two real or two complex solutions). Work some complicated or otherwise interesting examples; specifically including something like z^6=64. Use the ideas of magnitude and argument to graphically determine the six roots (two real, two complex conjugate pairs).

Closing

Time	Procedures
5m [5m total]	Assign homework; go over any immediately pressing questions.

Assessment (including diagnostic, formative and/or summative):
(Diagnostic/formative)
Questions / observations during class:

- During warm up, keep an eye out for students who don't finish quickly, as it's not meant to be a difficult problem. With their experience with complex numbers, it should be as easy as a normal quadratic.
- During first portion of instruction, students should be working out the Fundamental Theorem of Algebra. Questions are noted above, but to reiterate: How many roots will a quadratic have? A cubic? What about higher degrees? Why?
- Conjugate pairs are an important concept as to why the coefficients are real: students should be able to figure this out with working just a few examples of their own devising; likely allow students some time to work it out on their own. But same thing: How many complex/real roots can a quadratic have? Cubic? Higher degrees? Why?
- With the rotational problem (z^4=1 or something), ask students first to predict how many roots there will be, then how many of them will be complex.
- During the problem, have students graph it out and make connections using the graphical knowledge from previous lessons. Multiplying by itself k times to get magnitude whatever and k distinct angles.

(Formative)
Homework: a worksheet on polynomials with real coefficients and complex roots. Nothing too difficult, except for the challenge problems at the end for advanced students. Worksheet will be attached to this plan.

Additional Teacher Notes: