Name ______________________________________ Date ________________

Directions: Find all of the roots of each polynomial.

1) $x^2 + 16 = 0$

2) $x^2 + 6x + 18 = 0$

3) $x^4 = 16$

4) $x^2 - 4x - 12 = 0$

5) $x^2 + 6x + 13 = 0$

6) $x^3 - 3x^2 + 4x - 12 = 0$

Challenge Problems
Directions: These are optional bonus problems you may attempt if you desire.

C1) Rouché's Theorem states that for any two functions f and g that have no singularities in a certain region, if $|f| > |g|$ for all values on the boundary of the region, then f and g have the same number of zeroes in the region.
The proof of this theorem usually involves taking the inequality $|(f+g)-f| < |f-0|$ (true by |f| > |g|) and noting that the Euclidean distance between f+g and f must go to 0 when f=0, so f+g always has a zero when f has a zero, so g must also have a zero when f does.
When combined with a few more tools from complex analysis we don't yet have, this theorem provides a straightforward way to prove the Fundamental Theorem of Algebra.

Using Rouché's Theorem, can you prove that all the zeroes of $x^4 + 6x + 3$ have magnitude less than 2?
Hint: Consider $f = x^4$*, which we know has four zeroes (all at x=0) of magnitude less than 2.*