Name \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_

Directions: Find all of the roots of each polynomial.

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| 1) $x^{2}+16=0$ | 2) $x^{2}+6x+18=0$ |
|  |  |
| 3) $x^{4}=16$ | 4) $x^{2}-4x-12=0$ |
| 5) $x^{2}+6x+13=0$ | 6) $x^{3}-3x^{2}+4x-12=0$ |
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**Challenge Problems**

Directions: These are optional bonus problems you may attempt if you desire.

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| **C1)** Rouché’s Theorem states that for any two functions f and g that have no singularities in a certain region, if $\left|f\right|>|g|$ for all values on the boundary of the region, then f and g have the same number of zeroes in the region. The proof of this theorem usually involves taking the inequality $\left|\left(f+g\right)-f\right|<|f-0|$ (true by |f| > |g|) and noting that the Euclidean distance between f+g and f must go to 0 when f=0, so f+g always has a zero when f has a zero, so g must also have a zero when f does. When combined with a few more tools from complex analysis we don’t yet have, this theorem provides a straightforward way to prove the Fundamental Theorem of Algebra. Using Rouché’s Theorem, can you prove that all the zeroes of $x^{4}+6x+3$ have magnitude less than 2? *Hint: Consider* $f=x^{4}$*, which we know has four zeroes (all at x=0) of magnitude less than 2.*   |
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