Name \_Key\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_

Directions: Find all of the roots of each polynomial.

|  |  |
| --- | --- |
| 1)  | 2)  |
|  |  |
| 3)  | 4)  |
| Roots of unity. Can draw a graph to figure this out, or use the formula:  | (This is somewhat of a trick question – no complex numbers required)  |
| 5)  | 6)  |
|  |  |

**Challenge Problems**

Directions: These are optional bonus problems you may attempt if you desire.

|  |
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| **C1)** Rouché’s Theorem states that for any two functions f and g that have no singularities in a certain region, if for all values on the boundary of the region, then f and g have the same number of zeroes in the region. The proof of this theorem usually involves taking the inequality (true by |f| > |g|) and noting that the Euclidean distance between f+g and f must go to 0 when f=0, so f+g always has a zero when f has a zero, so g must also have a zero when f does. When combined with a few more tools from complex analysis we don’t yet have, this theorem provides a straightforward way to prove the Fundamental Theorem of Algebra. Using Rouché’s Theorem, can you prove that all the zeroes of have magnitude less than 2? *Hint: Consider , which we know has four zeroes (all at x=0) of magnitude less than 2.*   |
| Let and .On a circle of radius 2, .Similarly, on that circle, .Thus the hypotheses hold and f, g have the same # of zeroes in this region (four).Then f+g has all four of its zeroes in the same region as well by function addition. |