Name \_Key\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_

Directions: Find all of the roots of each polynomial.

|  |  |
| --- | --- |
| 1) | 2) |
|  |  |
| 3) | 4) |
| Roots of unity.  Can draw a graph to figure this out, or use the formula: | (This is somewhat of a trick question – no complex numbers required) |
| 5) | 6) |
|  |  |

**Challenge Problems**

Directions: These are optional bonus problems you may attempt if you desire.

|  |
| --- |
| **C1)** Rouché’s Theorem states that for any two functions f and g that have no singularities in a certain region, if for all values on the boundary of the region, then f and g have the same number of zeroes in the region.  The proof of this theorem usually involves taking the inequality (true by |f| > |g|) and noting that the Euclidean distance between f+g and f must go to 0 when f=0, so f+g always has a zero when f has a zero, so g must also have a zero when f does.  When combined with a few more tools from complex analysis we don’t yet have, this theorem provides a straightforward way to prove the Fundamental Theorem of Algebra.  Using Rouché’s Theorem, can you prove that all the zeroes of have magnitude less than 2?  *Hint: Consider , which we know has four zeroes (all at x=0) of magnitude less than 2.* |
| Let and .  On a circle of radius 2, .  Similarly, on that circle, .  Thus the hypotheses hold and f, g have the same # of zeroes in this region (four).  Then f+g has all four of its zeroes in the same region as well by function addition. |