Official Lesson Plan Form

Lesson Title: Complex Numbers in the z-plane   Date:

Subject/Class: Mathematics: Pre-Calculus   Grade Level: 11 / 12

MI Framework Standard(s) [(+) indicates an AP standard]:
N-CN:

2. Represent complex numbers and their operations on the complex plane.
    a. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
    b. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}\ i)$ has modulus 2 and argument 120°.
    c. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Objectives:
Upon completion of the lesson, students will be able to...
    1. Find the conjugate of a complex number and use it to help solve equations and convert forms.
    2. Find the modulus (or the magnitude) and the quotient (or the argument) of a complex number; convert from a magnitude and argument back to a complex number.
    3. Describe the correlation between vectors and complex numbers on the z-plane.
    4. Represent complex number arithmetic by vector combinations on the complex plane and convert between the two forms.
    5. Calculate the magnitude and argument of a complex number graphically.

Rationale for the Lesson:
In this lesson, students will be introduced to the concept of the complex plane and representing complex numbers graphically. This will be highly important in understanding why complex equations often have infinite solutions and in working far more easily with identities, as the plane provides several very useful ones. Additionally, viewing these numbers graphically helps students visualize highly abstract functions better, leading to better comprehension.

Student Prior Knowledge/Common Misconceptions:
Students should have an understanding of complex numbers in the form a+bi, as well as the basic operations +, -, *, / between various z (covered in previous lesson). Required prerequisite knowledge includes solid understanding of trig functions, polar form of real numbers, vectors, and graphing techniques (covered either earlier in this class or in a previous class).
One of the most common mistakes here is forgetting periodicity, where answers repeat due to the angle wrapping around again at 2pi. Students need to be comfortable with infinite numbers of solutions as a regular occurrence (ie solution at $z = \pi k, k \in \mathbb{Z}$.)

Materials and resources needed for lesson:

- Whiteboard, markers
- Worksheets / example problems (self-made if time allows)
- Paper/pencil
- Helpful: computer with graphing program installed such as MATLAB, Mathematica

References, when appropriate:

Opening

Time	Procedures/Details
5m 5m [10m total]	Warm up problem: Adding two vectors on the real plane. Vectors are in polar form (r, theta). Any questions from the homework?

Instructional Activities

Time	Procedures/Details
10m 10m 20m 10m 20m [70m total]	Introduce the form r(cos(theta)+isin(theta)) of a complex number and show how it's derived on a graph of the z-plane (use Matlab - or WolframAlpha if not available - to visualize). Demonstrate converting complex numbers between cartesian and polar form. Explain the magnitude and quotient (r and theta). Guide class to derive a formula for magnitude in a+bi form by observing r's relationship to a, b. Model a proof of this using academic language. Introduce the ideas of the four basic operations on polar-form complex numbers but do not give formulas. Form four groups and assign each one of the following tasks: 1) Find and prove a formula for r (or mag(z)) using the conjugate. 2) Find and prove a formula for adding/subtracting two complex numbers in polar form (either graphically or algebraically). 3) (2), but with multiplication. 4) (2), but with division. Help students out with formulas they may have forgotten (trig, especially – double angles and so on) and try to nudge them in the right direction. Work with them to ensure their proofs are using academic language and make sense. End of first day. Finish up group work from yesterday. Polish the proofs, ensure they're good to go. Collect homework from last lesson. Students present their proofs and demonstrate their formulas to the class. Each group turns in a copy of their proof to grade on correctness and academic language.

Closing

Time	Procedures
15m 5m [20m	Wrap up concepts the groups should have covered in their proofs. Ensure all basic operations on complex numbers (including +, -, *, /, and conjugation) make sense and students have at least two ways to do each, depending on the form. Apply these operations to determine distances and midpoints using standard formulas, but adapted for the complex plane. Assign homework for this lesson, go over it a little. Nothing too difficult, hopefully.

total]	

Assessment (including diagnostic, formative and/or summative):

(Diagnostic/formative)

Questions / observations during class:

- During warm up: hopefully students will remember their polar formulae from earlier; if not, we'll have to review a bit longer.
- While working with triangles, ask students to explain geometrically why these polar formulae work. Dig for the trigonometric basis underneath polar coordinates.
- After finishing modeling the proof, ask students to talk it over with their neighbors and see if it makes sense. Ensure they're aware they'll have to do something similar in a minute.
- Group students by which problem they'd like to solve. If you've got about equal numbers per group, great. Otherwise, second choices, third, etc.
- I don't expect students to remember complicated trig formulae needed for solving these problems; provide things like the double-angle formula in a cheat sheet, on the board, or something similar.
- Observe how professional / academic students' proofs are; I don't expect perfection, but I'd expect every claim to be justified. Work with covering edge cases, providing bases, etc.
- During review, ask students to come up with at least two ways of solving each problem. More if possible.

(Formative)

Homework: a worksheet on basic graphing and manipulation of polar-form complex numbers. Nothing too difficult, except for the challenge problems at the end for advanced students. Worksheet will be attached to this plan.

(Formative/summative)

Proofs / presentation:

Use same rubric as for big summative proof project, but with total point values reduced.

General guidelines: looking for students to understand and defend what they're presenting. Cover edge cases, generalize. Have either algebra or solid geometric reasons for their formulae (algebra preferred; during class, if they've settled on geometric, ask them how they can show the same result algebraically). Professional / academic language, grammar, etc.

Additional Teacher Notes: