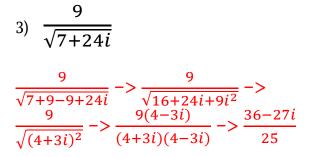
Name <u>Key</u>

Date _____

Directions: Convert each expression into a complex number in the form a + bi

1)
$$\frac{1+2i}{3-4i}$$

2) $\frac{5+2i}{\sqrt{3+4i}}$
 $\frac{(1+2i)(3+4i)}{(3-4i)(3+4i)}$
 $\frac{5+2i}{\sqrt{3+1-1+4i}} -> \frac{5+2i}{\sqrt{4+4i+i^2}} -> \frac{5+2i}{\sqrt{(2+i)^2}}$
 $\frac{-5+10i}{25}$
 $\frac{(5+2i)(2-i)}{(2+i)(2-i)} -> \frac{8-i}{3} -> \frac{8}{3} - \frac{i}{3}$
 $\frac{-1}{5} + \frac{2i}{5}$



4)
$$\left(1 + \frac{3}{1+i}\right)^2$$

 $\left(\frac{1(1+i)}{1+i} + \frac{3}{1+i}\right)^2 \longrightarrow \left(\frac{4+i}{1+i}\right)^2 \longrightarrow \frac{15+8i}{2i} \longrightarrow \frac{(15+8i)(-i)}{2i(-i)} \longrightarrow \frac{8-15i}{2} \longrightarrow 4 + \frac{-15i}{2}$

5)
$$\left(\frac{1+i}{1-i}\right)^4$$

6) $\left(\frac{1-i\sqrt{3}}{2+2i}\right)^2$
 $\left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^4 \rightarrow \left(\frac{(1-i)^2}{2}\right)^4 \rightarrow \left(\frac{-2i}{2}\right)^4$
 $\left(\frac{(1-i\sqrt{3})(2-2i)}{(2+2i)(2-2i)}\right)^2 \rightarrow \left(\frac{2-2\sqrt{3}-(2+2i\sqrt{3})}{8}\right)^2$
 $\rightarrow (-i)^4 = 1$
 $\rightarrow \frac{-\sqrt{3}}{4} + \frac{i}{4}$

Challenge Problems

Directions: These are optional bonus problems you may attempt if you desire.

C1) Prove that if z_0 is a root of the polynomial equation $z^n + a_1 z^{n-1} + a_2 z^{n-2} + \dots + a_n = 0$, with z = a + bi and $a_n \in \mathbb{R}$ (meaning that z is complex and all coefficients a_n are real), then $\overline{z_0}$ (the complex conjugate of z_0) is also a root of the equation. *Hint: Recall that the conjugate of a real number is the number itself. Hint: Recall that the product of two conjugates is the same as the conjugate of their product.*

We know: $\sum_{r=0}^{n} a_r z_0^{n-r} = 0$. We want: $\sum_{r=0}^{n} a_r \overline{z_0}^{n-r} = 0$. We know $\overline{z_0}^{n-r} = \overline{z_0^{n-r}}$ and $\overline{a_r} * \overline{z_0} = \overline{a_r} * \overline{z_0}$ as $\overline{z_1} * \overline{z_2} = \overline{z_1} * \overline{z_2}$. We know $\overline{a_r} = a_r$ as $a \in \mathbb{R}$. Hence: $\sum_{r=0}^{n} a_r \overline{z_0}^{n-r} = \sum_{r=0}^{n} \overline{a_r z^{n-r}} = \overline{\sum_{r=0}^{n} a_r z_0^{n-r}} = \overline{0} = 0$