Name Key

Date ______________

Directions: Convert each expression into a complex number in the form $a+bi$

1) $\frac{1+2i}{3-4i}$

$$\frac{(1+2i)(3+4i)}{(3-4i)(3+4i)}$$

$$\frac{-5+10i}{25}$$

$$\frac{-1}{5}+\frac{2i}{5}$$

2) $\frac{5+2i}{\sqrt{3+4i}}$

$$\frac{5+2i}{\sqrt{3+1-1+4i}} \text{ -> } \frac{5+2i}{\sqrt{4+4i+i^2}} \text{ -> } \frac{5+2i}{\sqrt{(2+i)^2}}$$

$$\frac{(5+2i)(2-i)}{(2+i)(2-i)} \text{ -> } \frac{8-i}{3} \text{ -> } \frac{8}{3}-\frac{i}{3}$$

3) $\frac{9}{\sqrt{7+24i}}$

$$\frac{9}{\sqrt{7+9-9+24i}} \text{ -> } \frac{9}{\sqrt{16+24i+9i^2}} \text{ -> }$$

$$\frac{9}{\sqrt{(4+3i)^2}} \text{ -> } \frac{9(4-3i)}{(4+3i)(4-3i)} \text{ -> } \frac{36-27i}{25}$$

4) $\left(1+\frac{3}{1+i}\right)^2$

$$\left(\frac{1(1+i)}{1+i}+\frac{3}{1+i}\right)^2 \text{ -> } \left(\frac{4+i}{1+i}\right)^2 \text{ -> } \frac{15+8i}{2i} \text{ -> }$$

$$\frac{(15+8i)(-i)}{2i(-i)} \text{ -> } \frac{8-15i}{2} \text{ -> } 4+\frac{-15i}{2}$$

5) $\left(\frac{1+i}{1-i}\right)^4$

$$\left(\frac{(1-i)^2}{(1+i)(1-i)}\right)^4 \text{ -> } \left(\frac{(1-i)^2}{2}\right)^4 \text{ -> } \left(\frac{-2i}{2}\right)^4$$

$$\text{-> } (-i)^4 = 1$$

6) $\left(\frac{1-i\sqrt{3}}{2+2i}\right)^2$

$$\left(\frac{(1-i\sqrt{3})(2-2i)}{(2+2i)(2-2i)}\right)^2 \text{ -> } \left(\frac{2-2\sqrt{3}-(2+2i\sqrt{3})}{8}\right)^2$$

$$\text{-> } \frac{-\sqrt{3}}{4}+\frac{i}{4}$$

Challenge Problems
Directions: These are optional bonus problems you may attempt if you desire.

C1) Prove that if z_0 is a root of the polynomial equation $z^n + a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n = 0$, with $z = a + bi$ and $a_n \in \mathbb{R}$ (meaning that z is complex and all coefficients a_n are real), then $\overline{z_0}$ (the complex conjugate of z_0) is also a root of the equation.
Hint: Recall that the conjugate of a real number is the number itself.
Hint: Recall that the product of two conjugates is the same as the conjugate of their product.

We know: $\sum_{r=0}^{n} a_r z_0^{n-r} = 0$. We want: $\sum_{r=0}^{n} a_r \overline{z_0}^{n-r} = 0$.
We know $\overline{z_0}^{n-r} = \overline{z_0^{n-r}}$ and $\overline{a_r} * \overline{z_0} = \overline{a_r * z_0}$ as $\overline{z_1} * \overline{z_2} = \overline{z_1 * z_2}$.
We know $\overline{a_r} = a_r$ as $a \in \mathbb{R}$.
Hence: $\sum_{r=0}^{n} a_r \overline{z_0}^{n-r} = \sum_{r=0}^{n} \overline{a_r z^{n-r}} = \overline{\sum_{r=0}^{n} a_r z_0^{n-r}} = \overline{0} = 0$ ■