Name \_Key\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Date \_\_\_\_\_\_\_\_\_\_\_\_

Directions: Convert each expression into a complex number in the form $a+bi$

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| --- | --- |
| 1) $\frac{1+2i}{3-4i}$ | 2) $\frac{5+2i}{\sqrt{3+4i}}$ |
| $$\frac{(1+2i)(3+4i)}{(3-4i)(3+4i)}$$$$\frac{-5+10i}{25}$$$$\frac{-1}{5}+\frac{2i}{5}$$ | $\frac{5+2i}{\sqrt{3+1-1+4i}}$ -> $\frac{5+2i}{\sqrt{4+4i+i^{2}}}$ -> $\frac{5+2i}{\sqrt{\left(2+i\right)^{2}}}$$\frac{(5+2i)(2-i)}{(2+i)(2-i)}$ -> $\frac{8-i}{3}$ -> $\frac{8}{3}-\frac{i}{3}$ |
| 3) $\frac{9}{\sqrt{7+24i}}$ | 4) $\left(1+\frac{3}{1+i}\right)^{2}$ |
| $\frac{9}{\sqrt{7+9-9+24i}}$ -> $\frac{9}{\sqrt{16+24i+9i^{2}}}$ -> $\frac{9}{\sqrt{\left(4+3i\right)^{2}}}$ -> $\frac{9(4-3i)}{(4+3i)(4-3i)}$ -> $\frac{36-27i}{25}$ | $\left(\frac{1(1+i)}{1+i}+\frac{3}{1+i}\right)^{2}$ -> $\left(\frac{4+i}{1+i}\right)^{2}$ -> $\frac{15+8i}{2i}$ -> $\frac{(15+8i)(-i)}{2i(-i)}$ -> $\frac{8-15i}{2}$ -> $4+\frac{-15i}{2}$ |
| 5) $\left(\frac{1+i}{1-i}\right)^{4}$ | 6) $\left(\frac{1-i\sqrt{3}}{2+2i}\right)^{2}$ |
| $\left(\frac{\left(1-i\right)^{2}}{(1+i)(1-i)}\right)^{4}$ -> $\left(\frac{\left(1-i\right)^{2}}{2}\right)^{4}$ -> $\left(\frac{-2i}{2}\right)^{4}$ -> $(-i)^{4}=1$ | $\left(\frac{\left(1-i\sqrt{3}\right)\left(2-2i\right)}{\left(2+2i\right)\left(2-2i\right)}\right)^{2}$ -> $\left(\frac{2-2\sqrt{3}-\left(2+2i\sqrt{3}\right)}{8}\right)^{2}$ -> $\frac{-\sqrt{3}}{4}+\frac{i}{4}$ |

**Challenge Problems**

Directions: These are optional bonus problems you may attempt if you desire.

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| **C1)** Prove that if $z\_{0}$ is a root of the polynomial equation $z^{n}+a\_{1}z^{n-1}+a\_{2}z^{n-2}+…+a\_{n}=0$, with $z=a+bi$ and $a\_{n}\in R$ (meaning that $z$ is complex and all coefficients $a\_{n}$ are real), then $\overline{z\_{0}}$ (the complex conjugate of $z\_{0}$) is also a root of the equation. *Hint: Recall that the conjugate of a real number is the number itself.**Hint: Recall that the product of two conjugates is the same as the conjugate of their product.* |
| We know: $\sum\_{r=0}^{n}a\_{r}z\_{0}^{n-r}=0$. We want: $\sum\_{r=0}^{n}a\_{r}\overline{z\_{0}}^{n-r}=0$. We know $\overline{z\_{0}}^{n-r}=\overline{z\_{0}^{n-r}}$ and $\overline{a\_{r}}\*\overline{z\_{0}}=\overline{a\_{r}\*z\_{0}}$ as $\overline{z\_{1}}\*\overline{z\_{2}}=\overline{z\_{1}\*z\_{2}}$. We know $\overline{a\_{r}}=a\_{r}$ as $a\in R$. Hence: $\sum\_{r=0}^{n}a\_{r}\overline{z\_{0}}^{n-r}=\sum\_{r=0}^{n}\overline{a\_{r}z^{n-r}}=\overline{\sum\_{r=0}^{n}a\_{r}z\_{0}^{n-r}}=\overline{0}=0$ $∎$ |